Dielectric Constant and Thickness Measurement of Dry Snowpack and Lake Icepack using Correlation Radiometry

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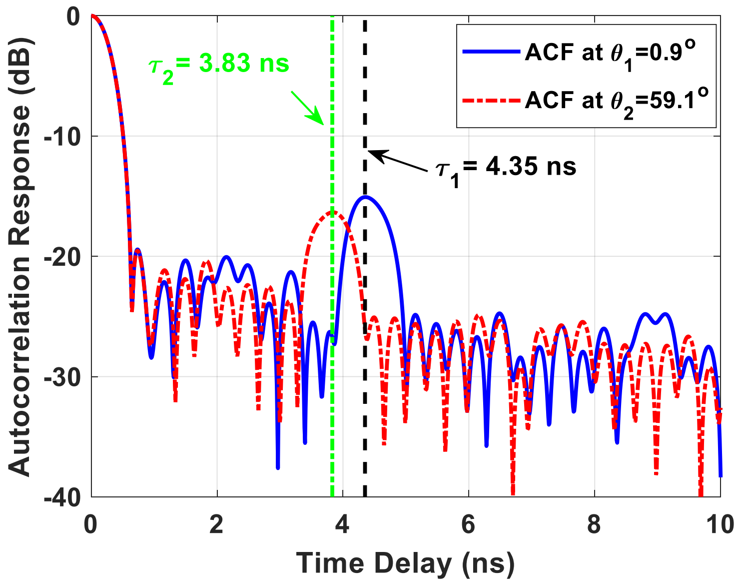
- Using the time delays measured by WiBAR at two distinct incidence angles:

$$\epsilon_{ice} = \frac{\tau_1^2 \sin^2\theta_2 - \tau_2^2 \sin^2\theta_1}{\tau_1^2 - \tau_2^2} = 3.24$$

$$d_{ice} = \frac{c\tau_1}{2\sqrt{\epsilon_p - \sin^2\theta_1}} = 36.24\ cm$$



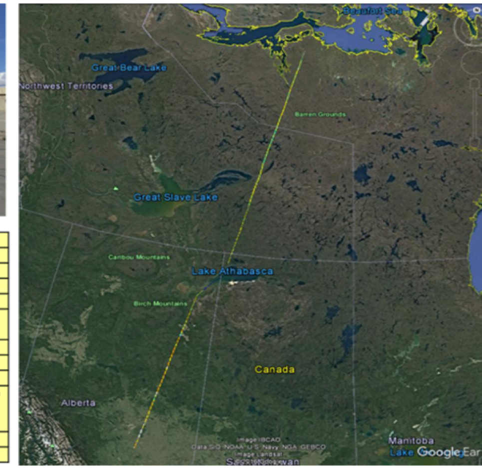
- The measured refractive index and thickness of the freshwater icepack by WiBAR was 3.24 and 36.24 cm ($\frac{\delta d}{d} \approx 2\%$, $\frac{\delta \epsilon_r}{\epsilon_r} \approx 1.8\%$). The ground truth was about 35.56 cm. The dielectric constant of a freshwater icepack $\epsilon_{ice} = 3.18$ over microwave frequencies (Matzler and Wegmuller[1]).

[1]C. Matzler and U. Wegmuller, ``The dielectric properties of fresh-water ice at microwave frequencies,'' Journal of Physics, D: Applied Physics, vol. 20, pp. 1623-1630, 14 Dec. 1987.

Future Plans

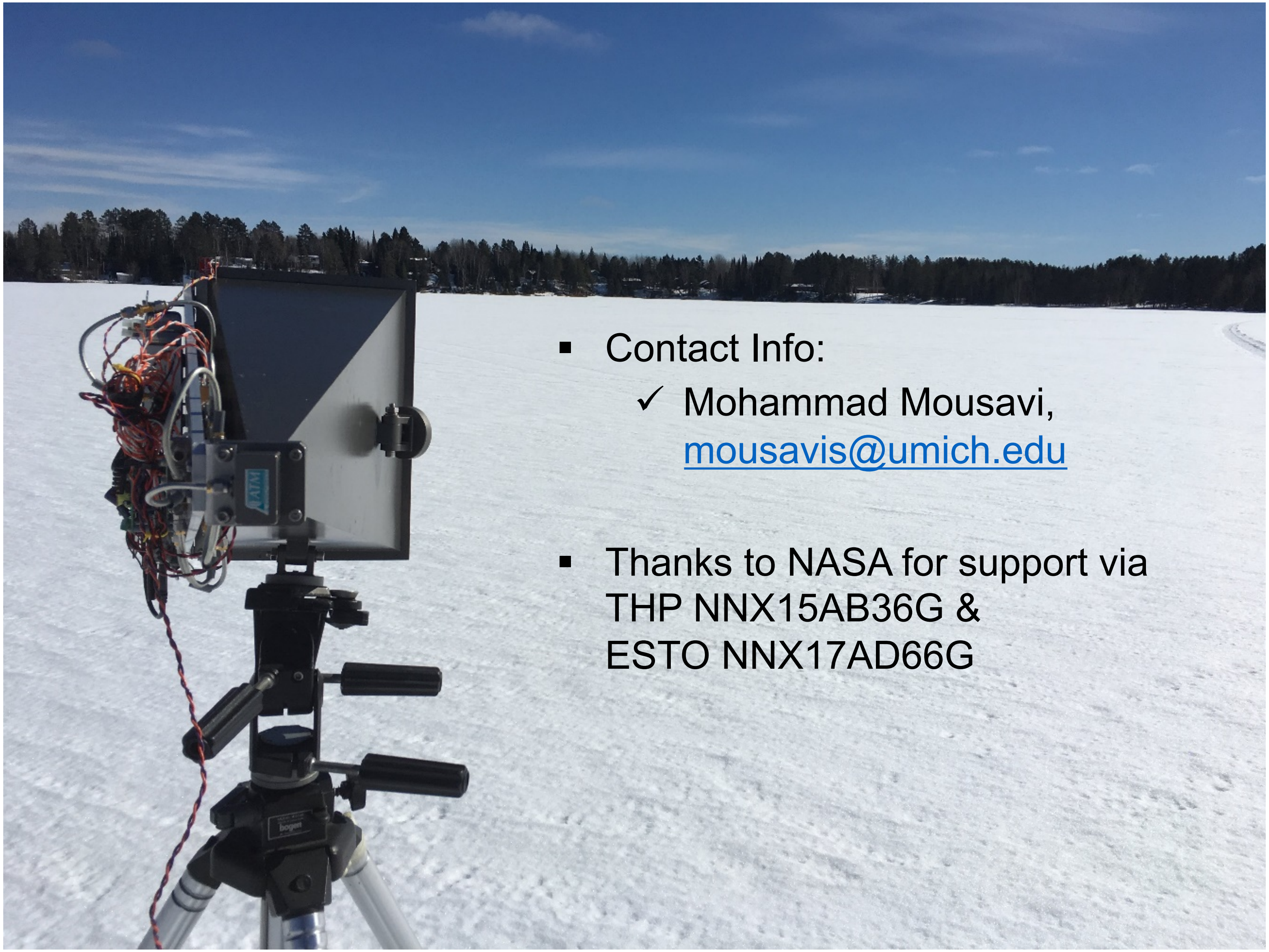
Sky is the limit!

- Ultra-Wideband Microwave Radiometer (UWBRAD)
  - The Ohio State University, Professor Joel Johnson's Group



Freq (GHz)	0.5-2, 12 x ~ 88 MHz channels
Polarization	Single (Right-hand circular)
Observation angle	Nadir
Spatial Resolution	1.2 km x 1.2 km (1 km platform altitude)
Integration time	100 msec
Ant Gain (dB) /Beamwidth	11 dB 60°
Calibration (Internal)	Reference load and Noise diode sources
Calibration (External)	Sky and Ocean Measurements
Noise equiv dT	0.4 K in 100 msec (each 88 MHz channel)
Interference Management	Full sampling of 88 MHz bandwidth in 16 bits resolution each channel; real time "software defined" RFI detection and mitigation
Initial Data Rate	700 Megabytes per second (10% duty cycle)
Data Rate to Disk	<1 Megabyte per second

- Finally from space!



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Error Analysis

Non-Destructive Dielectric Constant Measurement of a Loss-Less Dielectric Slab using FD-WiBAR

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- The lower bound on the variance of an unbiased estimator can be given by Cramer-Rao lower bound (CRLB):

$$Var_{\tau_{delay}}(N) \geq \frac{3\sigma_w^2}{(2\pi A_e\, \bar{e})^2\, N\,(N-1)(2N-1)}$$

where σ_w^2 is the variance of the added white Gaussian noise. If an estimator reaches this lower bound, it will be the minimum-variance unbiased (MVU) estimator.

- Since the time delay estimation using IFFT is a nonlinear problem, it will only reach the CRLB for signal to noise ratios (SNR) higher than a threshold value.

H. C. So, Y. T. Chan, Q. Ma and P. C. Ching, "Comparison of various periodograms for sinusoid detection and frequency estimation," in *IEEE Transactions on Aerospace and Electronic Systems*, vol. 35, no. 3, pp. 945-952, July 1999.

Non-Destructive Dielectric Constant Measurement of a Loss-Less Dielectric Slab using FD-WiBAR

- IFFT:

$$S[k] = \frac{1}{N}\sum_{n=0}^{N-1} e[n]\exp\left(i\frac{2\pi}{N}kn\right) \qquad e[n] = \bar{e}\left(1 - 2A_e\cos\left(2\pi f_n \tau_{delay}\right)\right)$$

- Denote the ACF coefficients at $\tau = \frac{k}{N}$ for the noise only and the signal present cases by $S_{e_0}(k)$ and $S_{e_1}(k)$, respectively. Hence:

$$S_{e_1}(k_0) = \sqrt{A^2 + B^2}$$

$$A \triangleq \frac{1}{N}\sum_{n=0}^{N-1}\left(\bar{e} - 2A_e\bar{e}\cos\left(2\pi n\tau_{delay}\right) + w[n]\right)\cos\left(\frac{2\pi n k_0}{N}\right)$$

$$B \triangleq \frac{1}{N}\sum_{n=0}^{N-1}\left(\bar{e} - 2A_e\bar{e}\cos\left(2\pi n\tau_{delay}\right) + w[n]\right)\sin\left(\frac{2\pi n k_0}{N}\right)$$

where A and B are independent Gaussian random variables and of identical variances of $\sigma^2 = \frac{\sigma_w^2}{2}$, where σ_w^2 is the power of the white Gaussian noise $w[n]$.

Non-Destructive Dielectric Constant Measurement of a Loss-Less Dielectric Slab using FD-WiBAR

- The PDF of $S_{e_1}(k_0)$, which is Ricean, is given by:

$$\rho_s(u) = \frac{u}{\sigma^2} \exp\left(-\frac{s^2 + u^2}{2\sigma^2}\right) I_0\left(\frac{u\, s}{\sigma^2}\right)$$

Where $s = A_e\, \bar{e}$ is the square root of the sum of the mean square of A and B, and I_0 is the modified Bessel function of the first kind of order zero.

- Similarly, the PDF of the remaining $S_{e_1}(k)$ and all the $S_{e_0}(k)$ are all of Rayleigh distribution:

$$\rho_w(u) = \frac{1}{\sigma^2} \exp\left(-\frac{u^2}{2\sigma^2}\right)$$

Non-Destructive Dielectric Constant Measurement of a Loss-Less Dielectric Slab using FD-WiBAR

- The mean square time delay error (MTDE) of the IFFT approach is given by:

$$MSTDE = (1-q)Var_{\tau_{delay}}(N) + q\int_0^{0.5} 2\left(u-\tau_{delay}\right)^2 du$$

where q is the probability of occurrence of an anomaly in the IFFT, as given by:

$$q = P\left(S_{e_1}(k_0) \le \text{at least one of the } S_{e_1}(1), S_{e_1}(2), \dots, S_{e_1}(k_0-1), S_{e_1}(k_0+1), \dots, S_{e_1}\left(\frac{N}{2}-1\right)\right)$$

$$= \int_0^{\infty} P\left(S_{e_1}(k_0)\right) \cdot \left[1 - \prod_{k=1,k\neq k_0}^{\left(\frac{N}{2}\right)-1} P\left(S_{e_1}(k) < u\right)\right] du$$

$$= \int_0^{\infty} \rho_S(u)\left[1 - \prod_{k=1,k\neq k_0}^{\left(\frac{N}{2}\right)-1} \int_1^2 \rho_W(v)dv\right] du$$

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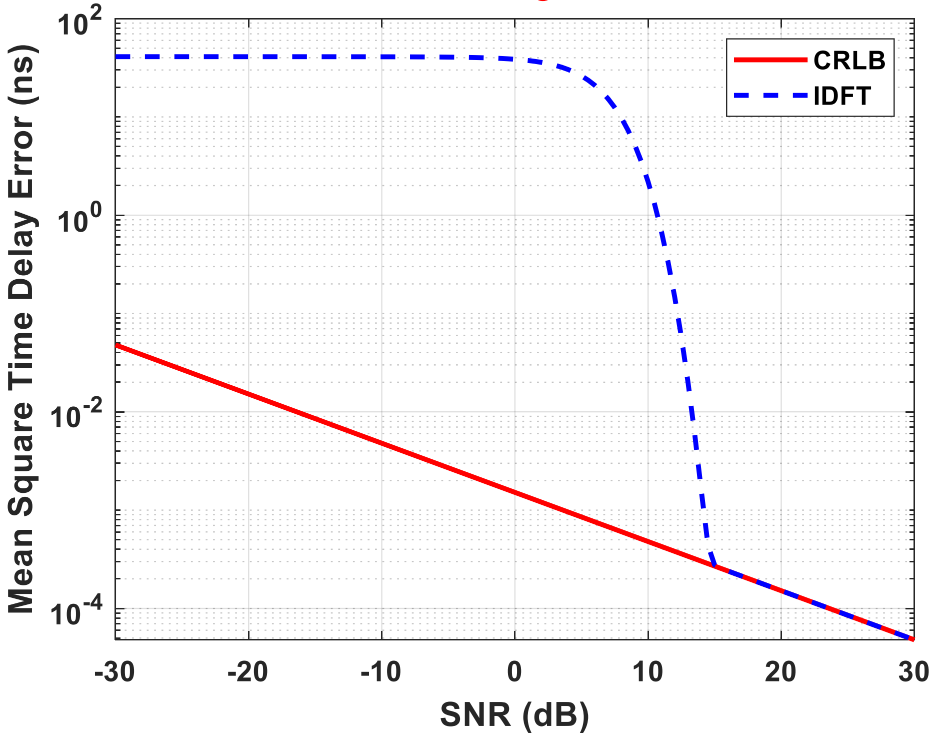
- If we define $SNR = \frac{\left(\frac{A_e e}{2}\right)^2}{\sigma_w^2}$, the CRLB would be:

$$Var_{\tau_{delay}}(N) \geq \frac{3}{16\pi^2\, N\,(N-1)(2N-1)SNR}$$

- Theoretical time delay variance of IFFT for an icepack with $\tau_{delay} = 3.83$ ns with respect to the SNR of the delay peak:

The incidence angle is $\theta = 0^o$.

The bandwidth is 3 GHz, and the number of points is $N = 461$.



Dielectric Constant and Thickness Measurement of Dry Snowpack and Lake Icepack using Correlation Radiometry

- Measured time delay by WiBAR at two distinct incidence angle:

$$\begin{cases} \tau_1 = \tau_{delay}(\theta_1) = \frac{2d_p}{c}\sqrt{\epsilon_p - \sin^2\theta_1} \\ \tau_2 = \tau_{delay}(\theta_2) = \frac{2d_p}{c}\sqrt{\epsilon_p - \sin^2\theta_2} \end{cases} \Rightarrow d_p \text{ and } \epsilon_p \text{ can be found}$$

- Assuming ϵ_p is real (Loss-Less pack):

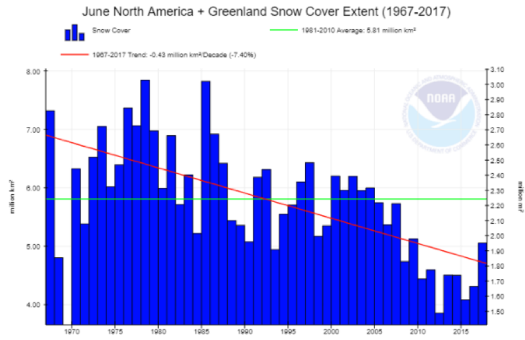
$$\epsilon_p = \frac{\tau_1^2 \sin^2\theta_2 - \tau_2^2 \sin^2\theta_1}{\tau_1^2 - \tau_2^2}$$

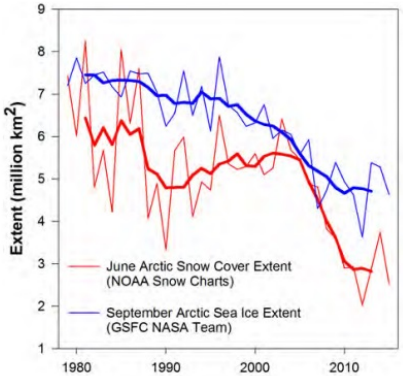
- Using measured ϵ_p:

$$d_p = \frac{c\tau_i}{2\sqrt{\epsilon_p - \sin^2\theta_i}}, \qquad i = 1, 2$$

Executive Summary

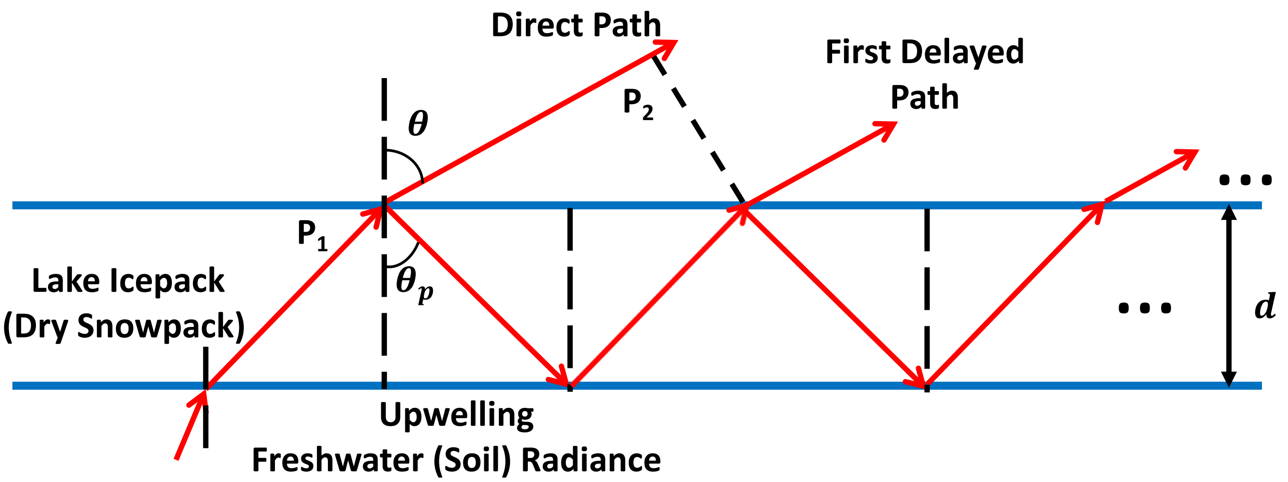
- **Objective:**
  - ➢ To remotely measure the snowpack depth, density, and thus snow water equivalent (SWE)
- **Importance of Snow and Ice:**
  - ➢ Snow is life!
- **Necessity of monitoring snow and ice:**
  - ➢ Significant changes in snow accumulation, timing, and melt
- **Method:**
  - ➢ Remotely sense the propagation time τ_{delay} of multi-path microwave emission of low-loss terrain covers at two distinct incident angles using a wideband autocorrelation radiometry (WiBAR)
- **Benefits:**
  - ➢ Passive microwave system
    - ✓ Low power (low cost) for space/air borne instruments
    - ✓ All weather operation capability
  - ➢ Deterministically measures the thickness and dielectric constant
    - ✓ No algorithm calibration needed
- **Challenges:**
  - ➢ Requires wide frequency bandwidth
  - ➢ Radio Frequency Interference (RFI) could be problematic

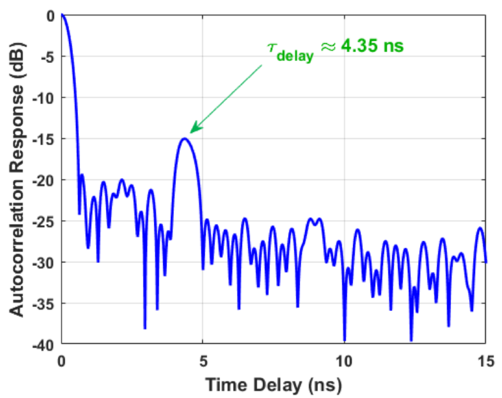




Sea ice and snow cover extents for the Northern Hemisphere from 1979 to 2015. The thick lines are 5-year running means (Derksen and Brown 2012).

Wideband Autocorrelation Radiometry (WiBAR)





This method (WiBAR) uses the coherence of the microwave emission from beneath the pack.

Integrated time delay:

$$\tau_{delay} = \frac{2}{c}\int_0^d \sqrt{\epsilon_p(z) - \sin^2\theta}\ dz$$

ϵ_p is the dielectric constant of the low-loss pack.